MATH3705 Tutorial 8

Determine the following:

1. $\mathcal{F}\left\{xe^{-3x^2}\right\}$.

   Solution:

$$\begin{aligned}\mathcal{F}\left(xe^{-3x^2}\right) &= \mathcal{F}\left(-\frac{1}{6}[e^{-3x^2}]'\right) = -\frac{1}{6}\mathcal{F}\left([e^{-3x^2}]'\right)\\ &= -\frac{1}{2}(-i\lambda)\mathcal{F}\left(e^{-3x^2}\right) = \frac{i\lambda}{2}\frac{\sqrt{\pi}}{\sqrt{3}}e^{-\lambda^2/12}.\end{aligned}$$

2. $\mathcal{F}\left\{(x+2)e^{-(x+2)^2}\right\}$.

   Solution: $\mathcal{F}\left\{(x+2)e^{-(x+2)^2}\right\} = e^{-2i\lambda}\mathcal{F}\left(xe^{-x^2}\right) = e^{-2i\lambda}\frac{i\sqrt{\pi}\lambda}{2}e^{-\lambda^2/4}$.

3. $\mathcal{F}\left\{xe^{-x^2+3ix}\right\}$.

   Solution: $\mathcal{F}\left\{xe^{-x^2+3ix}\right\} = \mathcal{F}\left\{e^{3ix}xe^{-x^2}\right\} = \frac{i\sqrt{\pi}(\lambda+3)}{2}e^{-(\lambda+3)^2/4}$.

4. $\mathcal{F}^{-1}\left\{e^{-\lambda^2}\right\}$.

   Solution: $\mathcal{F}^{-1}\left\{e^{-\lambda^2}\right\} = \frac{1}{2\sqrt{\pi}}e^{-x^2/4}$.

5. $\mathcal{F}^{-1}\left\{e^{2i\lambda-\lambda^2}\right\}$.

   Solution: $\mathcal{F}^{-1}\left\{e^{2i\lambda-\lambda^2}\right\} = \frac{1}{2\sqrt{\pi}}e^{-(x-2)^2/4}$.

6. $\mathcal{F}^{-1}\left\{\lambda e^{-\lambda^2}\right\}$.

   Solution: $\mathcal{F}^{-1}\left\{\lambda e^{-\lambda^2}\right\} = \frac{-ix}{4\sqrt{\pi}}e^{-x^2/4}$.

7. $\mathcal{F}^{-1}\left\{e^{-(\lambda+2)^2}\right\}$.

Solution: $\mathcal{F}^{-1}\left\{e^{-(\lambda+2)^2}\right\} = \frac{1}{2\sqrt{\pi}} e^{2ix-x^2/4}$.